14th Annual (International) Mechanical Engineering Conference – May 2006
Isfahan University of Technology, Isfahan, Iran

Modeling and Simulation of a Diesel Engine Common Rail Injector in Matlab/Simulink

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Abstract
In this paper a mathematical model of a Common Rail Injector (CRI) fitted to automotive diesel engines is presented. The CRI model includes submodels of solenoid, working chamber and needle. It has been simulated using Matlab/Simulink software environment. CRI is a fully flexible fuel injection system in which quantity, timing and pressure of the injection are controllable separately. Also, the non-linearity in fuel compression is considered. The high fuel pressure is supplied by a pump while an ECU calculates and sends a signal to injector which opens and shuts a solenoid. Sensitivity analysis of the model is also performed. The model results have been validated with experimental data.

Keywords: Common rail injector- Control solenoid – Computer simulation – Diesel engine – Fuel pressure.

Introduction
Diesel fuel injection system plays a vital role in diesel burning process by improving the atomization of the diesel fuel within the cylinder. Since diesel combustion is dependent on injection timing and mixing behavior, fuel injection at the right time with the right delivery and at a controlled injection rate are key factors in combustion and emission control. The direction towards solution of the problem is the creation of a highly effective fuel injectors based on latest technologies and electronic control. The characteristics of common rail injector (CRI) system make it a good candidate for diesel engine applications. In this work a detailed mathematical model of CRI is developed.
A number of CRI simulation models have been developed and published over the last few years. Gullaksen et al. [1] developed a computer program (written in Matlab) that implements the numerical solution of equation for plunger and needle dynamics and transient flow in high-pressure pipe line. Ahlin et al. [2] describe the behavior of pressure in the common rail diesel injection system mathematically. The model is base on well known physical relations.
Kiijarvi et al. [3] studied in detail the injection process of a medium speed diesel engine using a computer program for this purpose.
Wanhua Su et al. [4] developed a flexible injection rate common rail injector. Configuration of injector, fast response solenoid valve and design parameters of hydraulic system were presented through computer simulation and experimental data.
The aim of the present study is to build a one dimensional, transient and compressible flow model of the CRI. The model includes all electrical, mechanical and hydraulic subsystems and it is based on the Kirchhoff's law, the mass and momentum conservation equations and on the equilibrium of forces.
The model provides a powerful tool to evaluate the suitability of the CRI for diesel engine applications.

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2- PhD
3- PhD
4- PhD

The simulation of the model is implemented in Matlab/Simulink environment, which provides a higher degree of flexibility and allows simulation of linear and nonlinear elements.

Common Rail Injector

In this section the description of CRI is presented. Diesel engines exhaust particulates and NOx emissions are their significant drawbacks. The turning point in achieving better combustion characteristics is to provide higher injection pressures by the application of common rail injectors.

The high pressure fuel is supplied by a separate high pressure pump. An electronic control unit calculates and send a signal which opens and shuts a solenoid valve to obtain the required amount of fuel to be injected at the desired instant of time. More precise amount of injected fuel can be metered giving higher diesel efficiency. The fuel pressure is controlled by the pump and remains stable at a high pressure of approximately 160 MPa. High fuel pressure results in better fuel atomization, which leads to higher efficiency.

Figure 1 shows the section area of a Bosch injector [5]. This injector works as follows: The injector is connected directly to the high pressure fuel accumulator (rail) and the high pressure exists permanently at the needle seat chamber, which allows the injection timing to be controlled accurately. In order to control the opening and closing time of the needle, a small chamber (working chamber) with pressurized fuel is present on the top of the needle. The chamber is connected to the rail through a small orifice, which ensures that same pressure exists between the nozzle and the working chamber when the needle is closed. A solenoid valve, which receives an electrical signal at an specified time, is located on the top. When the solenoid valve is open, it creates a pressure drop in the working chamber so this will cause a negative force which overcomes the force of the needle spring and initiated the injection. As soon as the solenoid valve closes, the pressure in the working chamber rises again and forces the needle seat back.

System Identification

The system uses diesel oil as fuel. Fuel density heavily depends on pressure. It also depends on temperature but in a smaller scale. The fuel density will be calculated by a polynomial of the second order. It is assumed that density is only dependant on pressure.

It is assumed that all flows are turbulent and therefore the Reynolds number is bigger than 3000 for all flows. The flow coefficient for flow calculations in the model is taken as equal to 0.6.

Friction coefficient is a key factor in all machinery with moving parts. Since friction is very dependant on the contacting materials and the state of the interface, friction is measured under conditions as close to those of the operation as possible.

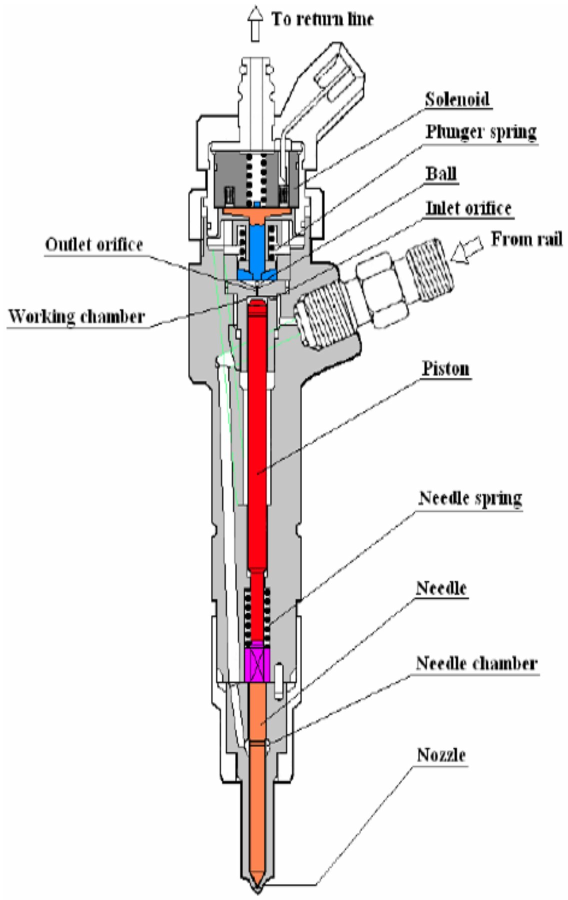

Fig. 1 – Diagram of a Bosch injector

Friction coefficient is taken as constant for all contacts inside the injector. Its value is assumed to be equal to 0.16 for well lubricated surfaces of the steel parts.

The system of units used in this study is SI.

Mathematical Model

Model of the solenoid valve

The solenoid receives an electrical signal from the ECU and opens ports in the CRI to begin the injection process. The solenoid must be extremely fast.

There are some assumptions for modeling of the solenoid:

- The coil inductance is assumed to be constant.
- The force of the pressurized fuel in the working chamber is assumed to be inversely proportional to the plunger movement.

- Any possible friction due to the bulging of the solenoid spring is ignored.

Based on standard magnetic circuit principles [6], the force of the solenoid can be expressed as:

$$F(x,i) = \frac{\mu_0 N^2 A i^2}{x^2} \tag{1}$$

Where; x is the plunger displacement, N is the number of turns of the solenoid coil, A is the cross-sectional area and μ_0 is permeability of free space.
The voltage equation can be expressed by:

$$e = \frac{d(L(x)\cdot i)}{dt} + R \cdot i + e_b \tag{2}$$

Where; L is the inductance of solenoid coil R is the solenoid coil resistance and e_b is the back e.m.f voltage.
This equation includes the phenomena of the back e.m.f voltage generated in the electrical circuit of the solenoid due to the movement of the plunger. Back e.m.f voltage is:

$$e_b = K_b \frac{dx}{dt} \tag{3}$$

Where; K_b is the back e.m.f coefficient of the solenoid.
The force generated by the solenoid is proportional to the current via the force constant:

$$F(x,i) = K_f(x) \cdot i \tag{4}$$

Where; K_f is the force constant of the solenoid.
For the purpose of modeling, polynomials of the fourth order are applied to describe the dependence of the force of the solenoid on the displacement of the plunger. So the current of the solenoid circuit is determined from following differential equation:

$$\frac{di}{dt} = \frac{1}{L}\left(e - R \cdot i + \frac{F(x,i)}{i} \cdot \frac{dx}{dt}\right) \tag{5}$$

Movement of the solenoid plunger for injector is modeled by the following equation:

$$m_1 \frac{d^2x}{dt^2} = K_f i + P_L S_1 - m_1 g - k_1(x + x_0) - f_1 \frac{dx}{dt} \tag{6}$$

Where m_1 is the mass of solenoid plunger, x_{max} is the maximum plunger movement, P_L is the pressure in the working chamber, S_1 is the cross-sectional area of the outlet orifice, k_1 is the stiffness of the solenoid plunger spring, g is the gravitational acceleration and f_1 is the friction coefficient between the plunger and the stationary parts of the solenoid.

Model of the Hydraulic components

For the modeling of the hydraulic components the following assumptions are applied:

- Speed of pressure propagation is infinite.
- Fuel leakage between the components of the injector is ignored.
- The outlet orifice's cross sectional area is proportional to $\frac{x}{X_{max}}$.
- The pipe wall expansion is not taken into calculation.
- It is assumed that no air is present in the fuel.

With these assumptions, a one dimensional, transient and compressible flow model of the hydraulic component of the CRI is built base on the mass conservation and force equilibrium equations. These equations describe flows through an orifice and take into consideration the fuel compressibility. The fuel flow through the outlet orifice when the solenoid valve is opening and closing causes a flow out through outlet orifice, acts on the fuel volume and moves the piston system.
Hence the mass conservation equation for the working chamber can be expressed as follows [7]:

$$S_2 \frac{dy}{dt} - \frac{V_{wc}}{\beta}\frac{dP_W}{dt} = K_1 S_1 \sqrt{\frac{2(P_L - P_W)}{\rho}} - K_0 S_0 \frac{x}{X_{max}} \sqrt{\frac{2(P_W - P_R)}{\rho}} \tag{7}$$

Where V_{wc} is the volume of the working chamber, P_W is the pressure in the working chamber, P_L is the pressure in the line, P_R is the pressure in the return line, S_0 is the cross-sectional area of the inlet orifice, S_2 is the cross-sectional area of the piston, K_1 is the outlet flow coefficient and K_0 is the inlet flow coefficient.
In this equation the fuel bulk modulus β is a function of pressure, which is selected especially for diesel oil [3].
The force acting on the piston system is the force of the pressurized fuel in the needle chamber. This overcomes the force of needle spring, the force of the pressurized fuel in the working chamber, the force due to friction and gravity force. The actuating

force causes the piston system to move upwards, open the nozzle and provide an injection.
By applying the Newton seconds' law (force equilibrium) to the injector piston, the piston system motion equation is:

$$m_2 \frac{d^2y}{dt^2} = P_L S_N - P_W S_2 - k_2(y + y_0) - m_2 g - f_2 \frac{dy}{dt} \tag{8}$$

Where m_2 is the weight of piston system, y is the piston movement, y_0 is the pre-compressed spring movement, s_N is the effective cross-sectional area of the needle, k_2 is the spring force coefficient and f_2 is the friction coefficient in the system piston-liner.
Based on the "flow through an orifice" theory, the injection flow rate is calculated by the following equation [8]:

$$q_I = K_I S_I \frac{y}{Y_{\max}} \sqrt{\frac{2(P_L - P_W)}{\rho}} \tag{9}$$

Where; K_I is the flow coefficient through the nozzles, S_I is the effective nozzle area, and $Y_{\max}$ is the maximum movement of the needle.
The actual effective nozzle area is directly proportional to $\frac{y}{Y_{\max}}$, i.e. the injection flows through changing orifices, which is equal $S_I \frac{y}{Y_{\max}}$.
Because the needle chamber of the CRI is directly connected to the high-pressure accumulator, the line pressure is assumed to be constant during injection.
Lastly, the injection quantity is the integral of the injection flow arte for every shot:

$$Q = \int_0^t q_I \, dt \tag{10}$$

Model Implementation

Based on the equations presented in the previous section, the model of the CRI was implemented in Matlab/Simulink software. Simulink program provides a more intuitive understanding of the physical phenomenon than does a set of mathematical equations.
Figure 2 shows the block diagram of the model of CRI. This model consists of four individual blocks namely: Solenoid, Needle, Upper chamber and Force equilibrium. It allows independent evaluation and modification of individual blocks without changing the structure of the whole model.

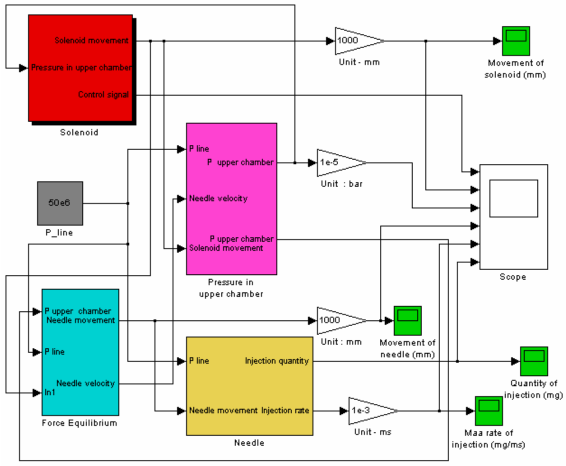

Fig.2 – Common Rail Injector model

The non-linearities and discontinuities of the system are resolved by placement of "Matlab Fcn" blocks. These blocks are functions written in Matlab. The Matlab/Simulink environment also allows easily modification of parameters and introduction of their dependence on pressure.
The advantage of this structure is that it allows observation of inputs and outputs and easy debugging of program.
The observed results could provide a base for further justification of a development in the injector's modifications.

Model Validation

Measurements have performed for the solenoid control pulses of 15V DC with the durations of 2 and 4 ms. The results of measurements together with simulated responses are shown. These figures show a very good match between the experimental and simulated results.
The quantities of injections were also compared. Experimental data are obtained from reference [9]. Table 1 presents the results received from the experimental and simulated tests:

Table 1 – Comparison of the injection quantities

Duration of Pulses(ms)	Number Of Injections	Pressure (bar)	Total Quantity (g)
4	100	500	11.4
6	100	500	17.2
8	100	500	22.7
10	50	500	14.2

Average Measured Quantity Of an Injection(mg)	Simulated Quantity Of an Injection (mg)	Error (%)
114	115.819	1.6
172	169.017	-1.7
227	222.402	-2.0
284	275.825	-2.8

The results show that the CRI model, with a good degree of accuracy, can be used for the investigation of possible use of the injector for pilot injection.

Simulation Results

The investigations of the model were performed to observe the system behavior. This is a special investigation of the injector's sensitivity, which represent a very important feature of a Common Rail Injector and are indicative of the speed and accuracy the injector can perform its function.

Figures representing the output curves developed from a 5 ms control signal are shown.

Figure 3 shows the square-shape control signal of the model which is the voltage of solenoid electrical circuit. Figure 4 represents the movement of solenoid core curve. It is directly dependent on the shape of the control signal. Figure 5 shows the variations of working chamber pressure. When the needle is closed, fuel pressure in this chamber is about 500 bars. As the needle starts to open, pressure decreases to the amount of 40 bars at the point where the needle is fully opened. Figure 6 shows the movement shape of the needle. As it is seen in the figure, needle movement is similar in shape with solenoid movement. Figure 7 shows the injection rate diagram. It is proportional to needle movement. Figure 8 shows the injection quantity curve. This quantity is the integral of injection rate during the injection phase.

We are interested in the investigation of the sensitivity of major output parameters of the CRI injector. Which are:

- Injection delay
- Injection duration
- Injection quantity

Using the developed model we have performed sensitivity analysis of these parameters for following input parameters:

- Force coefficient of the spring of the solenoid (k_1)
- Force coefficient of the spring of the needle (k_2)
- Diameter of the piston (d_p). The diameter here understood as a complex parameter affecting the values of other dependant parameters. These include the cross-sectional area of the piston, the mass of the piston and the volume of the working chamber.
- Temperature of the coil of the solenoid (T_s).

The results of the sensitivity analysis are shown in table 2:

Table 2 – Sensitivity analysis of the CRI model

	k_1	k_2	d_p	T_s
Injection Delay	-0.173	0.085	0.69	1.6
Injection Duration	-0.049	-0.051	-0.05	-0.512
Injection Quantity	-0.061	-0.059	-0.068	-0.638

The results show that the system is relatively insensitive to changes in the force coefficients of the solenoid and needle springs, but it is very sensitive to the solenoid temperature and the possible design change in the overall diameter of the piston. According the model investigation, the change in the diameter of the piston, which consequently causes changes in the related parameters, may significantly affect the injection delay. The positive sensitivity of 0.69 points out that a possible dimension minimization may reduce the injection delay. The sensitivity analysis shows that the temperature of the solenoid coil must be taken into calculations.

If the injection system requires a shorter time interval then a faster solenoid will be needed [10].

Conclusion

The study presents a one-dimensional, transient and compressible flow model of a common rail injector. Based on electrical, magnetic and fluid hydraulic equations, the model represent complete dynamic behavior of diesel injector. The significant feature of the model, compared to existing computer models, is as follows:

The model of a solenoid is present as a submodel. The injector is electrically controlled, and electromechanical device connects the electrical, mechanical and hydraulic parts of the injector.

Dimension of the piston of the CRI is critical and in the case of pilot injection the speed of the solenoid is the major limiting factor of the performance.

The developed model provides better understanding of the issues and limitations of the injector. It gives detailed insight into their working principles. A number of injector parameters, which are difficult to measure in practice, can be observed during simulation.

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Figures:

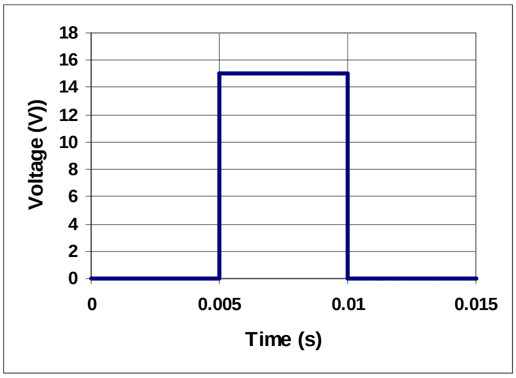

Fig.3 – Control signal (voltage)

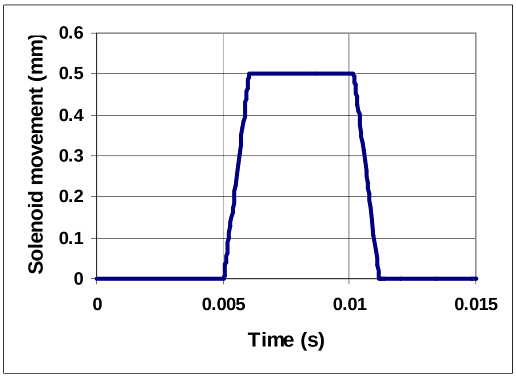

Fig.4 – Solenoid movement (mm)

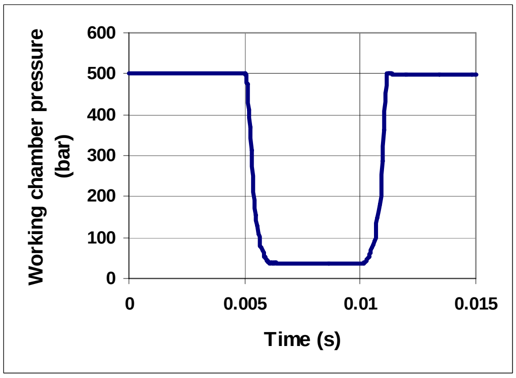

Fig.5 – Pressure of working chamber (bar)

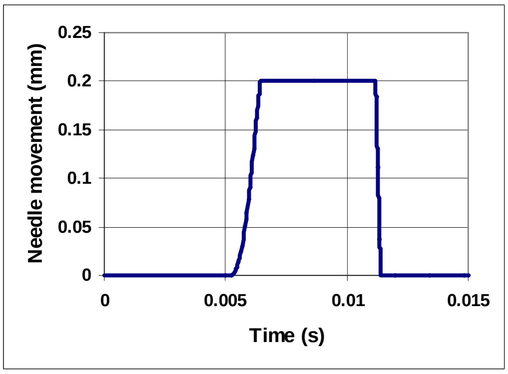

Fig.6 – Movement of needle (mm)

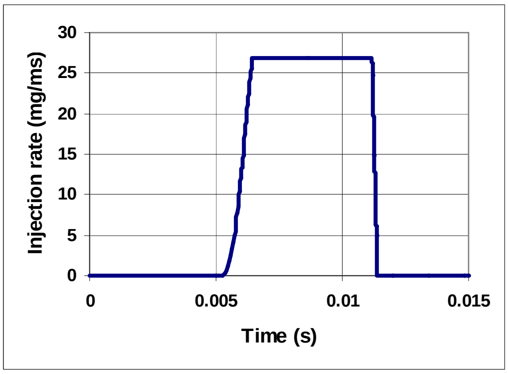

Fig.7 – Rate of the injection (mg/ms)

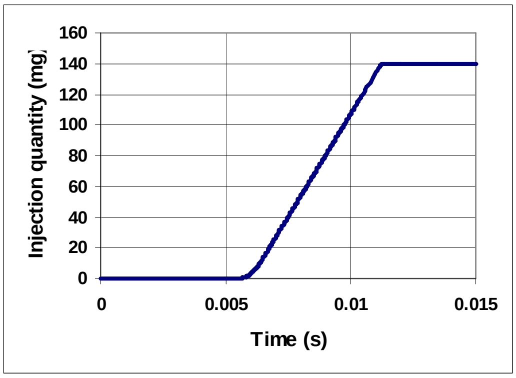

Fig.8 – Quantity of injection (mg)